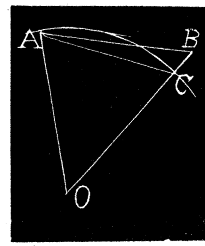
$$\therefore AC = \frac{BC\cos(AOC-34')}{\sin(\frac{1}{2}AOC-34')} \ldots\ldots \ldots\ldots\ldots\ldots (1).$$



$$AB = \frac{BC\cos\frac{1}{2}AOC}{\sin(\frac{1}{2}AOC-34')} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2).$$

$BC=1$ from (1). $\quad 2R\sin\frac{1}{2}AOC = \dfrac{\cos(AOC-34')}{\sin(\frac{1}{2}AOC-34')}$.

$\therefore \cos(AOC-34') = \dfrac{(1+R)\cos 34'-1}{R+1}$. $\therefore$ $AOC = 1°58'16''$. From (2) $AB = 136.778$ miles.

Errata. On page 56, second line from top, for "$-2562Z^3$" read $-256Z^3$; sixth line from top for "$Z=2.750413$" read $Z=2.750458368$, and for "$WR=1.5248$" read $WR=1.2963390864$; and in Note, second line from bottom, for "$z=WR$" read $x=WR$.

PROBLEMS.

38. Proposed by S. H. WRIGHT, M. D., A. M., Ph. D., Penn Yan, New York.

In latitude $42°30'$ north$=\lambda$, at what angle with the horizon, will the sun rise, its declination$=22°$ north$=\delta$?

39. Proposed by SETH PRATT, C. E., Assyria, Michigan.

The pendulum of a clock which gains 6 seconds in 1 hour and 13 minutes, makes 6000 vibrations in 1 hour and $9\frac{1}{2}$ minutes. What is the length of the pendulum? And what length should it have to keep true time?

QUERIES AND INFORMATION.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

The Origin of π.—At least from the time of Archimedes π has stood for the number expressing how many times the diameter the circumference is. It is the initial letter of the Greek word περιφέρεια, meaning periphery. If the diameter is taken as a unit, then π stands for the periphery, or circumference. This is in reply to query of Lottie Smith in December Number.

Benj. F. Yanney,
Mount Union College, Alliance, Ohio.

An Expression for π.—Though the result is not new, I have not seen it developed as follows:

$$\text{Since } e^{i\pi}=-1, \quad \therefore\ i\pi\log e=\log(-1).$$

$$\therefore\ \pi=\frac{\log(-1)}{\sqrt{-1}}.$$

BENJ. F. YANNEY.

Referring to the Note of R. Greenwood in the December Number, I would state that (1) probably the other root was infinite. Thus the equations $x^2-y^2=5$ and $x+y=5$ have roots $x=3$ or ∞, and $y=2$ or $-\infty$. (2) The proof that imaginary roots enter in pairs assumes that all the coefficients are real. The equation $x^2-bix=a^2-abi$ has roots: a and $-a+bi$ but its coefficients are not all real. (3) The equation $\sqrt{2x_2-2}-(3x-5)=0$ or A must be multiplied by $\sqrt{2x^2-2}+(4x-5)=B$ or 0 to give a quadratic equation. The given equation is not of the second degree as Mr. Greenwood seems to imply but of the $\frac{2}{2}$ degree. An infinite number of equations can be written that have no roots at all: for instance, $2x-5+\sqrt{x^2-7}=0$ (or ?). This when combined with its congeneric $2x-5-\sqrt{x^2-7}=?$ or 0 gives a quadratic; the last expression takes both roots leaving no root for the first form. The demonstration that every equation has a root referred to equations free from surds. H. C. WHITAKER,
Manual Training School, Philadelphia.

Another Reply. By squaring the equation, we get $2x^2-2=(3x-5)^2\ldots(1)$ $=2x^2-2-(3x-5)^2=0$, which is equivalent to transposing the member $3x-5$, and then multiplying the equation by $\sqrt{2x^2-2}+(3x-5)=0$. By doing this we have really introduced a new equation, which is satisfied for $x=\frac{9}{7}$.

Observe that (1) is satisfied for both $x=3$, and $x=\frac{9}{7}$, for it contains both the original equation and the one introduced by the questionable operation of squaring. Therefore, if the given equation means the positive root of $(2x^2-2)=3x-5$, then 3 is the only value of x that will satisfy it.

If $\pm\sqrt{2x^2-2}=3x-5$, both 3 and $\frac{9}{7}$ will.

BENJ. F. YANNEY,
Mount Union College, Alliance, Ohio.

Note on Solution IV., Page 190. It does not follow that triangies AEL and ADK are equal because the triangles AEL and ADK are similar respectively to AFN and AGM, and the solution fails.

I would like to see a direct proof of this problem. It is said that the mathematician Todhunter failed to produce a direct proof of it.

GEORGE LILLEY,
394 Hall Street, Portland, Oregon.

Problem by Euler. One answer is given, but he adds there are many more. Legendre asks for a general solution as Euler's solution is lost: and he says such

a solution would be very much prized by mathematicians, if it could be given.

1st. The sum of the squares of each, horizontal, vertical, or diagonal rows shall be equal,—10 conditions.

A	B	C	D
E	F	G	H
I	K	L	M
N	O	P	Q

68	-29	41	-37
-17	31	79	-32
59	28	23	61
-11	-77	8	49

=Euler's Numbers.

2nd. The sum of their products taken two and two=0, taking any two rows, horizontal, vertical, and diagonals,—12 conditions.

$$AE+BF+CG+DH=0=AD+FG+KL+NQ, \text{ etc.}$$

Hillsboro, Ill., Mathematical Club.

Note on No. 4—Miscellaneous. In regard to No. 4 Miscellaneous, I had worked the problem with the assumption made by Prof. Hume, but rejected my solution, as on further thought I did not consider the assumption warranted. The constantly changing curvature carries with it a change in actual contact as well as in the amount ground off, which I have not been able to analyze. The assumption made would seem to apply if the stones were kept pressed together with such a force as would not yield, and would cause the particles to overlap for a constant distance. This also would require a constantly changing pressure or adjustment.

I should like to ask whether any one knows of a principle which will apply to the effect of friction in a case of this kind.

C. W. M. Black,
Wesleyan Academy, Wilbraham, Mass.

Query. Is a man who writes for publication in a Mathematical Magazine a "Note on Helmholtz's use of the terms 'Surface' and 'Space' as identical in meaning", properly to be considered sane?

Again when he asks "Does the 'immortal' Helmholtz in his Lectures on the—'Origin and Significance of Geometrical Axioms'—use the terms 'surface' and 'space' as identical in meaning?" since Helmholtz never delivered any lectures under this title, would it be sane to attempt to answer?

G. B. Halsted.

The equation from Bell's Algebra, quoted by Mr. Greenwood, (MONTHLY, Vol., p. 372) is consistent if the radical be given the double sign. The equation should be

$$\pm\sqrt{2x^2-2}=3x-5.$$

The value $x=3$ belongs to the upper sign, $x=\frac{9}{7}$ to the lower.

WM. E. HEAL.

The answer to query (Monthly, Vol. II., p. 247) is not satisfactory. It is true "We *have* no method of finding the cube root by means of a compass" [and rule] but that does not prove the *impossibility* of a solution. What I wish, is a rigorous proof of the impossibility of expressing the roots of a cubic equation by a geometrical construction.

WM. E. HEAL.

Concerning the value of *factorial zero*, Chrystal says (Text Book of Algebra, Part II., page 4) "Strictly speaking 0! has no meaning. It is convenient, however, to use it, with the understanding that its value is 1; by so doing we avoid the exceptional treatment of initial terms in many series."

WM. E. HEAL.

IS THERE MORE THAN ONE ILLIMITABLE SPACE?

The Metageometers assume without proof that there are many varieties of space, differing in curvature, in the number of dimensions and in extent. Is their assumption axiomatic or does it need proof? Is it not really inconsistent with the hypothesis that space is everywhere and illimitable?

The Metageometers concede that the space that contains our Universe may for aught they know to the contrary, be trinally extended, i. e., through any point of it, whatever, three straight lines may be drawn mutually at right angles to each other. Notwithstanding this concession, they assume that there are two varieties of space at least, the number of whose dimensions is less than three.

They call a surface a variety of space that has *two* dimensions, and a line a variety of space that has *one* dimension.

The Euclidian geometers locate all their lines and surfaces in the one, trinally extended, illimitable space. They do not regard these lines and surfaces as distinct varieties of space that may be classed under an n-fold species.

Some of the Metageometers call a line one dimensional space, and a surface two dimensional space, apparently with the expectation that this ambiguous use of the word space will somehow assist them in ascending from our tridimensional space to a hypothetical one of four dimensions, and from that to one of five dimensions, and so on. This is certainly a most hazardous enterprise that they have undertaken. They are attempting to scale the transcendental heights of Hyper-space with an analogical ladder constructed out of defective timber. The two bottom rounds—one dimensional space and two dimensional space—are unable to endure the strain put upon them. We do not mount to trinally extended space from surfaces, nor to surfaces from lines. But we start with trinally extended space and in it locate surfaces and lines.

Successful ascent cannot be made from tridimensional space to fourdimensional space.

1st.—Because no one knows or can know the direction from 3-fold space to 4-fold, even if the latter exists.

2nd.—Because no one knows or can know that 4-fold space exists for the reason that the fundamental laws of thought are violated in every effort of the mind to cognize it. Legitimate thinking cannot proceed in violation of logical law, but stultification may do so. The so-called "generalized space" of the Metageometers is believed to be the joint product of pseudo-generalization, pseudo-analogical reasoning, and pseudo-analytical interpretation.

JOHN N. LYLE.

BOOKS AND PERIODICALS.

Trigonometry for Schools and Colleges. By Frederick Anderegg, A. M., Professor of Mathematics, and Edward Drake Roe, Jr., A. M., Associate Professor of Mathematics in Oberlin College. 8vo. Cloth, 108 pp. Boston : Ginn & Co.

This little work is a decided improvement over most modern treatises on trigonometry. It treats the subject with clearness and accuracy and leads the student to an easy acquaintance with modern higher mathematics. A number of new features are introduced. This is the first book we have yet seen in which it is shown that Plane Trigonometry is a special case of Spherical Trigonometry. Many other subjects of equal interest and importance are discussed. The authors deserve much credit for this original and unique work.

B. F. F.

An Elementary Treatise on Rigid Dynamics. By W. J. Loudon, B. A., Demonstrator in Physics in the University of Toronto. 8vo. Cloth, 236 pp. Price, $2.25. New York : Macmillan & Co.

This is a most excellent treatise on Rigid Dynamics. The subjects treated are made very clear and the student is still further aided in grasping those complex and difficult principles by very beautiful and accurate diagrams. Any student who has mastered the calculus can take up this work without any difficulty. At the close of each subject is a list of problems. The book closes with 306 problems all of which are very interesting to the student of dynamics. Some of these excellent problems will appear in future numbers of the MONTHLY.

B. F. F.

Notations de Logique Mathématique. Par G. Peano, Professeur d'Analyse infinitésimale à l'Université de Turin. Introduction au Formulaire de Mathématique Publie par la *Revista di Matematico*, Turin. Pamphlet, 52 pages.

A very interesting and valuable treatment of the notations of mathematical logic.

B. F. F.